

Linear Algebra

Elements of Matrices, Determinants,
Inverse of Matrices

Elements of a Matrix

$$A = \begin{bmatrix} 3 & 1 \\ -2 & 4 \\ -1 & 6 \end{bmatrix}$$

Elements

3, 1, -2,
4, -1, 6

Order

3 × 2

↑ ↑
Number Number
of Rows of Columns

2 × 3 **X**

**Position of
an element**

(Row , Column)

(1, 1)th element → 3

(1, 2)th element → 1

⋮

(3, 2)th element → 6

Addition and Subtraction

$$A = \overset{2 \times 2}{\begin{bmatrix} 3 & -4 \\ 8 & 5 \end{bmatrix}} \quad B = \overset{2 \times 2}{\begin{bmatrix} 9 & -5 \\ 2 & -6 \end{bmatrix}}$$
$$A + B = \begin{bmatrix} 3 + 9 & -4 + (-5) \\ 8 + 2 & 5 + (-6) \end{bmatrix}$$
$$A + B = \begin{bmatrix} 12 & -9 \\ 10 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 7 & -2 \\ 8 & 3 \end{bmatrix} \quad D = \begin{bmatrix} 6 & -1 \\ 5 & 3 \end{bmatrix}$$

$$C - D = \begin{bmatrix} 7 - 6 & -2 + (+1) \\ 8 - 5 & 3 - 3 \end{bmatrix}$$

$$C - D = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 8 & -5 \\ 4 & 6 & 2 \\ -7 & 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 3 & 7 \\ -2 & 8 & 4 \\ 3 & -5 & -9 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3+4 & 8+3 & -5+7 \\ 4+(-2) & 6+8 & 2+4 \\ -7+3 & 1+(-5) & 3+(-9) \end{bmatrix}$$

$$A + B = \begin{bmatrix} 7 & 11 & 2 \\ 2 & 14 & 6 \\ -4 & -4 & -6 \end{bmatrix} =$$

$$E = \begin{bmatrix} 2 & 5 & -9 \\ 4 & -6 & 3 \end{bmatrix} \quad F = \begin{bmatrix} 7 & 3 & -6 \\ 5 & -8 & -4 \end{bmatrix}$$

$$E - F = \begin{bmatrix} 2-7 & 5-3 & -9+(-6) \\ 4-5 & -6+(-8) & 3+(-4) \end{bmatrix}$$

$$E - F = \begin{bmatrix} -5 & 2 & -3 \\ -1 & -14 & -1 \end{bmatrix}$$

Multiplication

$$A = \begin{matrix} & \overset{1}{1} & \overset{2}{4} & \overset{3}{-2} \\ \underset{1}{1} & & & \\ \underset{2}{2} & & & \end{matrix} \begin{bmatrix} 1 & 4 & -2 \\ 3 & 5 & -6 \end{bmatrix} \quad B = \begin{matrix} & \overset{1}{5} & \overset{2}{2} & \overset{3}{8} & \overset{4}{-1} \\ \underset{1}{1} & & & & \\ \underset{2}{2} & & & & \\ \underset{3}{3} & & & & \end{matrix} \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix}$$

$$AB = \begin{matrix} & \overset{1}{2} & \overset{2}{1} \\ \underset{1}{1} & & \\ \underset{2}{2} & & \end{matrix} \begin{bmatrix} 2 & 1 \\ & \end{bmatrix}$$

$$\boxed{1(5) + 4(3) + (-2)(-2)}$$

$$5 + 12 + 4 = 17 + 4 = \boxed{21}$$

$$A = \begin{matrix} & \overset{1}{1} & \overset{2}{4} & \overset{3}{-2} \\ \underset{1}{1} & & & \\ \underset{2}{2} & & & \end{matrix} \begin{bmatrix} 1 & 4 & -2 \\ 3 & 5 & -6 \end{bmatrix} \quad B = \begin{matrix} & \overset{1}{5} & \overset{2}{2} & \overset{3}{8} & \overset{4}{-1} \\ \underset{1}{1} & & & & \\ \underset{2}{2} & & & & \\ \underset{3}{3} & & & & \end{matrix} \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix}$$

$$AB = \begin{matrix} & \overset{1}{2} & \overset{2}{1} & \overset{3}{8} \\ \underset{1}{1} & & & \\ \underset{2}{2} & & & \end{matrix} \begin{bmatrix} 2 & 1 & 8 \\ & & \end{bmatrix}$$

$$1(2) + 4(6) + (-2)(9)$$

$$2 + 24 - 18 = 26 - 18 = 8$$

$$A = \begin{bmatrix} 1 & 4 & -2 \\ 3 & 5 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 21 & 8 & 10 \end{bmatrix}$$

$$1(8) + 4(4) + (-2)(7)$$

$$8 + 16 - 14 = 8 + 2 = 10$$

$$A = \begin{bmatrix} 1 & 4 & -2 \\ 3 & 5 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 21 & 8 & 10 & 25 \end{bmatrix}$$

$$1(-1) + 4(5) + (-2)(-3)$$

$$-1 + 20 + 6 = -1 + 26 = 25$$

$$A = \begin{matrix} & \overset{1}{1} & \overset{2}{4} & \overset{3}{-2} \\ \underset{2}{2} & \begin{bmatrix} 1 & 4 & -2 \\ 3 & 5 & -6 \end{bmatrix} \end{matrix} \quad B = \begin{matrix} & \overset{1}{5} & \overset{2}{2} & \overset{3}{8} & \overset{4}{-1} \\ \underset{2}{2} & \underset{3}{3} & \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix} \end{matrix}$$

$$AB = \begin{matrix} & \overset{1}{21} & \overset{2}{8} & \overset{3}{10} & \overset{4}{25} \\ \underset{2}{2} & \begin{bmatrix} 21 & 8 & 10 & 25 \\ 42 & & & \end{bmatrix} \end{matrix}$$

$$3(5) + 5(3) + (-6)(-2)$$

$$15 + 15 + 12 = 30 + 12 = 42$$

$$A = \begin{matrix} & \overset{1}{1} & \overset{2}{4} & \overset{3}{-2} \\ \underset{2}{2} & \begin{bmatrix} 1 & 4 & -2 \\ 3 & 5 & -6 \end{bmatrix} \end{matrix} \quad B = \begin{matrix} & \overset{1}{5} & \overset{2}{2} & \overset{3}{8} & \overset{4}{-1} \\ \underset{2}{2} & \underset{3}{3} & \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix} \end{matrix}$$

$$AB = \begin{matrix} & \overset{1}{21} & \overset{2}{8} & \overset{3}{10} & \overset{4}{25} \\ \underset{2}{2} & \begin{bmatrix} 21 & 8 & 10 & 25 \\ 42 & -18 & & \end{bmatrix} \end{matrix}$$

$$3(2) + 5(6) + (-6)(9)$$

$$6 + 30 - 54 = 6 - 24 = -18$$

$$A = \begin{matrix} & \overset{1}{1} & \overset{2}{4} & \overset{3}{-2} \\ \underset{1}{1} & & & \\ \underset{2}{2} & & & \end{matrix} \begin{bmatrix} 1 & 4 & -2 \\ 3 & 5 & -6 \end{bmatrix} \quad B = \begin{matrix} & \overset{1}{5} & \overset{2}{2} & \overset{3}{8} & \overset{4}{-1} \\ \underset{1}{1} & & & & \\ \underset{2}{2} & & & & \\ \underset{3}{3} & & & & \end{matrix} \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix}$$

$$AB = \begin{matrix} & \overset{1}{21} & \overset{2}{8} & \overset{3}{10} & \overset{4}{25} \\ \underset{1}{1} & & & & \\ \underset{2}{2} & & & & \end{matrix} \begin{bmatrix} 21 & 8 & 10 & 25 \\ 42 & -18 & 2 & \end{bmatrix}$$

$$3(8) + 5(4) + (-6)(7)$$

$$24 + 20 - 42 = 44 - 42 = 2$$

$$A = \begin{matrix} & \overset{1}{1} & \overset{2}{4} & \overset{3}{-2} \\ \underset{1}{1} & & & \\ \underset{2}{2} & & & \end{matrix} \begin{bmatrix} 1 & 4 & -2 \\ 3 & 5 & -6 \end{bmatrix} \quad B = \begin{matrix} & \overset{1}{5} & \overset{2}{2} & \overset{3}{8} & \overset{4}{-1} \\ \underset{1}{1} & & & & \\ \underset{2}{2} & & & & \\ \underset{3}{3} & & & & \end{matrix} \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix}$$

$$AB = \begin{matrix} & \overset{1}{21} & \overset{2}{8} & \overset{3}{10} & \overset{4}{25} \\ \underset{1}{1} & & & & \\ \underset{2}{2} & & & & \end{matrix} \begin{bmatrix} 21 & 8 & 10 & 25 \\ 42 & -18 & 2 & 40 \end{bmatrix}$$

$$3(-1) + 5(5) + (-6)(-3)$$

$$-3 + 25 + 18 = 22 + 18 = 40$$

$$\begin{aligned}
 A &= \begin{matrix} & \overset{1}{1} & \overset{2}{4} & \overset{3}{-2} \\ \underset{1}{1} & \begin{bmatrix} 1 & 4 & -2 \\ 3 & 5 & -6 \end{bmatrix} \\ \underset{2}{2} & \end{matrix} & B &= \begin{matrix} & \overset{1}{5} & \overset{2}{2} & \overset{3}{8} & \overset{4}{-1} \\ \underset{1}{1} & \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix} \\ \underset{2}{2} & \underset{3}{3} & \end{matrix} \\
 & \quad \quad \quad \textcircled{2} \times \textcircled{3} & & \textcircled{3} \times \textcircled{4} \\
 AB &= \begin{matrix} & \overset{1}{21} & \overset{2}{8} & \overset{3}{10} & \overset{4}{25} \\ \underset{1}{1} & \begin{bmatrix} 21 & 8 & 10 & 25 \\ 42 & -18 & 2 & 40 \end{bmatrix} \\ \underset{2}{2} & \end{matrix} \\
 & \quad \quad \quad 3(-1) + 5(5) + (-6)(-3) \\
 & \quad \quad \quad -3 + 25 + 18 = 22 + 18 = 40
 \end{aligned}$$

Determinant of a Matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

$$\begin{bmatrix} 3 & 5 \\ -4 & 7 \end{bmatrix} = 3(7) - 5(-4) \\ 21 + 20 = 41$$

$$\begin{bmatrix} -7 & 8 \\ 4 & -3 \end{bmatrix} = -7(-3) - 4(8) \\ = 21 - 32 \\ = -11$$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \\ = a_1 \begin{bmatrix} b_2 & c_2 \\ b_3 & c_3 \end{bmatrix} - b_1 \begin{bmatrix} a_2 & c_2 \\ a_3 & c_3 \end{bmatrix} + c_1 \begin{bmatrix} a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & -3 \\ 5 & 7 & 6 \\ -8 & 1 & 9 \end{bmatrix} = 114 - 372 - 183 \\ = -441$$

$$= 2 \begin{bmatrix} 7 & 6 \\ 1 & 9 \end{bmatrix} - 4 \begin{bmatrix} 5 & 6 \\ -8 & 9 \end{bmatrix} + -3 \begin{bmatrix} 5 & 7 \\ -8 & 1 \end{bmatrix}$$

$$= 2[7(9) - 1(6)] - 4[5(9) - (6)(-8)] - 3[5(-8) - (-56)]$$

$$= 2[63 - 6] - 4[45 + 48] - 3[61]$$

$$= 2(57) - 4(93) - 183$$

$$\begin{aligned}
 & \begin{bmatrix} 5 & 7 & -8 \\ 4 & 3 & 6 \\ 1 & 7 & -9 \end{bmatrix} \\
 &= 5 \begin{bmatrix} -3 & 6 \\ 7 & -9 \end{bmatrix} - 7 \begin{bmatrix} 4 & 6 \\ 1 & -9 \end{bmatrix} + -8 \begin{bmatrix} 4 & -3 \\ 7 & 7 \end{bmatrix} \\
 &= 5(27 - 42) - 7[-36 - 6] - 8[28 - -3] \\
 &= 5(-15) - 7[-42] - 8[31] \\
 &= -75 + 294 - 248 = \boxed{-29}
 \end{aligned}$$

$$\begin{aligned}
 & \begin{bmatrix} 1 & 0 & 4 & 6 \\ 2 & 5 & 0 & 3 \\ -1 & 2 & 3 & 5 \\ 2 & 1 & -2 & 3 \end{bmatrix} = +1 \begin{bmatrix} 5 & 0 & 3 \\ 2 & 3 & 5 \\ 1 & -2 & 3 \end{bmatrix} - 0 \begin{bmatrix} 2 & 0 & 3 \\ -1 & 3 & 5 \\ 2 & -2 & 3 \end{bmatrix} + 4 \begin{bmatrix} 2 & 5 & 3 \\ -1 & 2 & 5 \\ 2 & 1 & 3 \end{bmatrix} - (-6) \begin{bmatrix} 2 & 5 & 0 \\ -1 & 2 & 3 \\ 2 & 1 & -2 \end{bmatrix} \\
 &= 74 - 0 + 4(52) + 6(6) \\
 &= 74 + 208 + 36 \\
 &= 318
 \end{aligned}$$

$$\begin{array}{r}
 11 \\
 208 \\
 74 \\
 + 36 \\
 \hline
 318
 \end{array}$$

Inverse of a Matrix A

Defining the Inverse Matrix

numbers

$$x \cdot \frac{1}{x} = 1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

matrices

$$\mathbf{A} \mathbf{A}^{-1} = \mathbf{I}$$

$$\mathbf{A}^{-1} \mathbf{A} = \mathbf{I}$$

identity
matrix

Finding the Inverse of a Two-By-Two Matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

- 1) a and d **swap positions**
- 2) b and c **switch their sign**
- 3) **divide** by the determinant

Finding the Inverse of a Two-By-Two Matrix

$$A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} \quad A^{-1} = \frac{1}{-1} \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$$

$$A A^{-1} = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

the **identity matrix** confirms
this inverse relationship

 Utilizing Inverse Matrices

solving equations with matrices
 (inverse matrices act as matrix division)

$$\frac{XA}{A} = \frac{B}{A}$$

this is not possible

 Utilizing Inverse Matrices

solving equations with matrices
 (inverse matrices act as matrix division)

$$XA A^{-1} = B A^{-1}$$

 Utilizing Inverse Matrices

solving equations with matrices
 (inverse matrices act as matrix division)

$$X I = B A^{-1}$$

**a matrix times its inverse
 yields an identity matrix**
 (like cancelling out the matrix)

Finding the Inverse of a Three-By-Three Matrix

Step 1: Generate the Matrix of Minors

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

each entry is a particular **determinant**

Step 1: Generate the Matrix of Minors

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -24 & & \\ & & \\ & & \end{bmatrix}$$

$$(1)(0) - (4)(6) = -24$$

Step 1: Generate the Matrix of Minors

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -24 & -20 & \\ & & \\ & & \end{bmatrix}$$

$$(0)(0) - (4)(5) = -20$$

Step 1: Generate the Matrix of Minors

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -24 & -20 & -5 \\ & & \\ & & \end{bmatrix}$$

$$(0)(6) - (1)(5) = -5$$

Finding the Inverse of a Three-By-Three Matrix

Step 1: Generate the Matrix of Minors

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -24 & -20 & -5 \\ & & \\ -18 & & \end{bmatrix}$$

$$(2)(0) - (3)(6) = -18$$

Step 1: Generate the Matrix of Minors

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -24 & -20 & -5 \\ -18 & -15 & \\ & & \end{bmatrix}$$

$$(1)(0) - (3)(5) = -15$$

Step 1: Generate the Matrix of Minors

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -24 & -20 & -5 \\ -18 & -15 & -4 \\ & & \end{bmatrix}$$

$$(1)(6) - (2)(5) = -4$$

Step 1: Generate the Matrix of Minors

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -24 & -20 & -5 \\ -18 & -15 & -4 \\ 5 & & \end{bmatrix}$$

$$(2)(4) - (3)(1) = 5$$

Finding the Inverse of a Three-By-Three Matrix

Step 1: Generate the Matrix of Minors

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -24 & -20 & -5 \\ -18 & -15 & -4 \\ 5 & & \end{bmatrix}$$

$$(2)(4) - (3)(1) = 5$$

Step 1: Generate the Matrix of Minors

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -24 & -20 & -5 \\ -18 & -15 & -4 \\ 5 & 4 & 1 \end{bmatrix}$$

$$(1)(1) - (2)(0) = 1$$

Step 1: Generate the Matrix of Minors

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -24 & -20 & -5 \\ -18 & -15 & -4 \\ 5 & 4 & 1 \end{bmatrix}$$

matrix of minors

Step 2: Generate the Matrix of Cofactors

$$\begin{bmatrix} -24 & -20 & -5 \\ -18 & -15 & -4 \\ 5 & 4 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

matrix of cofactors

Finding the Inverse of a Three-By-Three Matrix

Step 2: Generate the Matrix of Cofactors

$$\begin{bmatrix} -24 & -20 & -5 \\ -18 & -15 & -4 \\ 5 & 4 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} -24 & - & -5 \\ - & -15 & - \\ 5 & - & 1 \end{bmatrix}$$

matrix of cofactors

corner and center entries
will remain as they are

Step 3: Find the Adjugate/Adjoint

$$\begin{bmatrix} -24 & -20 & -5 \\ -18 & -15 & -4 \\ 5 & 4 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix}$$

adjugate/adjoint

we must reflect entries
across the main diagonal

Step 4: Divide Adjugate by Determinant

$$\begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix} \div \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

adjugate/adjoint original matrix

Step 4: Divide Adjugate by Determinant

$$\begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix} \div \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

adjugate/adjoint

$$1 \begin{vmatrix} 1 & 4 \\ 6 & 0 \end{vmatrix} - 2 \begin{vmatrix} 0 & 4 \\ 5 & 0 \end{vmatrix} + 3 \begin{vmatrix} 0 & 1 \\ 5 & 6 \end{vmatrix} = -24 + 40 - 15 = 1$$

Finding the Inverse of a Three-By-Three Matrix

Step 4: Divide Adjugate by Determinant

$$\begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix} \div 1$$

adjugate/adjoint

divide every entry in the
adjugate by this **determinant**

Step 1: Find the Matrix of Minors

(each entry is a particular determinant)

Step 2: Find the Matrix of Cofactors

(follow the plus/minus checkerboard)

Step 3: Find the Adjugate

(transpose entries across the main diagonal)

Step 4: Divide by Determinant

(divide each entry by determinant of original matrix)

Step 4: Divide Adjugate by Determinant

$$\begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix}$$

this is our final answer
for the **inverse matrix**